Dynamics of two particles in a plasma sheath

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The stability and arrangements of two dust particles in a plasma are investigated in terms of the Hamiltonian of the system. It is shown that the Hamiltonian description of a non-Hamiltonian system can be used to predict qualitative features of possible equilibria in a variety of confinement potentials and can provide useful plasma diagnostics. The results compare favorably with those of simulations and are used to create experimental hypotheses. In particular, the symmetry-breaking transition of the particles as they leave the horizontal plane admits a Hamiltonian description which is used to elucidate the wake parameters.

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I. INTRODUCTION

Plasma systems containing small numbers of microparticles (i.e., dust) have attracted theoretical as well as experimental interest due the complexity of the interactions involved and their application in the study of so-called nonextensive systems [1] containing long-range interactions comparable to the size of the system. Plasmas containing dust can also serve as a useful model for many-particle systems in such diverse areas as condensed matter physics, astrophysics, and atmospheric physics.

Complex plasmas [1–3] provide an ideal medium for studying structural transitions in nonextensive systems, when even the system of two particles displays rich physics [4–7]. For example, the binding energy resulting from the attractive wake potential created by two particles in streaming plasma [8] causes them to behave as a molecule [9]. The study of this dust molecule has revealed useful insights into larger plasma crystals, including the vertically aligned crystal structure and the driving force behind crystal-liquid phase transitions.

The nature of particle arrangements in systems containing large numbers of particles can be understood by considering simplified systems of just a few (e.g., two) particles. It is well known that even an interaction between two dust particles in the plasma sheath environment is highly complicated. The interaction involves both a symmetrical Debye-type interaction and an asymmetric attractive interaction caused by plasma collective processes. The stationary charged dust particles perturb the trajectories of vertically flowing ions in the sheath toward a focus underneath the particles, creating a region of enhanced positive space charge in their wake. The theory of wake formation has been described [10–14] and verified by experiments [15] as well as three-dimensional particle-in-cell and molecular dynamics simulations [16,17].

In addition to the vertical sheath field in which the dust particles levitate against the force of gravity [1], a radial electric field is typically imposed to trap the particles within the discharge. Under varying discharge parameters, the particles have been found to align either parallel or perpendicular to the direction of ion flow [4–6]. The stability of the particle arrangements is determined by a combination of confinement strengths, interparticle forces and wake effects [7]. The structure and dynamics of dust arrangement in flowing plasma with constant and uniform ion velocity were considered by Lampe *et al.* [18], where formation of stable self-bound molecules was demonstrated and possible equilibria dust pairs confined in quadratic (parabolic) and quartic external potentials were reported.

In this paper, we focus on the symmetry-breaking disruptions that can occur in a system of two dust particles when the key parameters (such as confinement, dust charge, separation, and ion flux) have reached their critical values. We discuss the possible equilibria of the dust system in a variety of confinement potential profiles. The analysis suggests that the parabolic potential well admits only horizontal and vertical stable dust alignments which is in agreement with experiments and a recent theoretical prediction [18]. It is also suggested that continuous symmetry breakings of the dust particle alignments are possible not just with a quadratic potential well, but also with a cubic anharmonicity, due to particle asymmetry and spatial variation of the grain charge with elevation in the sheath. It is shown that the horizontal alignment instability at the critical confinement provides a diagnostic tool for determining the wake parameters.

Although the dust dynamics in complex plasma has been proven to be non-Hamiltonian due to nonreciprocal wake effects, friction against stationary neutrals, and spatial charge variations [19], it is shown here that the Hamiltonian can be used to make qualitative predictions about the existence and stability of the equilibria in this system. The extent to which Hamiltonian formalism can be used to model the dynamics of a non-Hamiltonian system is one of the general questions we attempt to address.

II. THE MODEL

The potential part of the Hamiltonian $\mathcal{H}=T+V$ can be separated into an interparticle potential $V_{\rm int}$ and an interaction energy with the external confinement $V_{\rm conf}$. In order to determine the interparticle coupling energy, it is necessary to find the potential of the dust particles. The potential of an isolated dust particle of charge Q_d immersed in a plasma flowing along the z axis is given by

$$\Phi(\boldsymbol{r}) = \int \frac{Q_d}{2^2k^2}\frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}}}{\varepsilon(\boldsymbol{k},k_z v_t)}d\boldsymbol{k},$$

where v_t is the velocity of the test dust particle relative to the plasma (ion) flow, ε is the plasma dielectric response, and

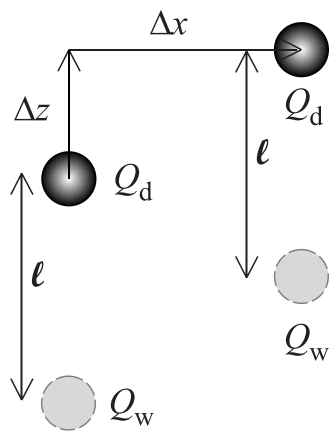

FIG. 1. Physical dimensions of the two-particle system used in the model.

$\boldsymbol{k}=(k_{\perp},k_z)$ is the position in Fourier space. This can be separated as $\Phi=\Phi_D+\Phi_w$ [11,13] where $\Phi_D=(Q_d/4\epsilon_0 r)e^{-\kappa r}$ is the usual Debye potential for a screened point charge, Φ_w is the potential due to the wakefield behind the dust particle and $\kappa=1/\lambda_D$ is the inverse screening length. In the recent study [20], an analytical expression was derived for the potential of a test charge in a weakly ionized plasma with ion drift. The expression obtained was shown to agree with experimental measurements in the sheath. In one of the simplest models [7], one assumes that the wake field is always attractive and takes the form of a positive point potential located at a fixed distance ℓ beneath the dust grain (see Fig. 1),

$$\Phi_w(\rho,z)=\frac{Q_w}{4\epsilon_0}\frac{e^{-\kappa\sqrt{\rho^2+(z+\ell)^2}}}{\sqrt{\rho^2+(z+\ell)^2}}.$$

Note that the wake-particle interaction is communicated only one way and therefore cannot be included in the Hamiltonian description of the system. In the general case for Hamiltonian approximation, the potential depends on the particle positions x_1,x_2,z_1,z_2. For the two-particle case, due to the symmetry of the system, the potential depends only on the interparticle separations,

$$V=\frac{Q_d^2}{4\epsilon_0\Delta_d}e^{-\kappa\Delta_d}+V_{\mathrm{conf}}(\Delta x,\Delta z), \tag{1}$$

where $\Delta_d=\sqrt{\Delta x^2+\Delta z^2}$ is the interparticle distance. In the absence of dissipative forces we may claim that the dust molecule resides in one of the minima of (1).

When the radial confinement is the weak and the wake is not too strong, the potential can have absolute minima, with the particles horizontally aligned. As radial confinement increases, the potential energies of the horizontally aligned minima increase, causing them to become only local minima, with the absolute minima residing in the vertical alignment. If the horizontal confinement is sufficiently strong, the horizontal equilibria become only saddle points and the only stable equilibria lie in vertical alignment. Note that the conclusion that the only equilibria are either horizontal or vertical, is obvious for a pair of particles in a cylindrically symmetric but not spherically symmetric confining potential, interacting via any isotropic interaction force when oblique equilibria are possible. Moreover, oblique equilibria are also possible for cylindrically symmetric biquadratic potentials as well. This was first demonstrated by Lampe *et al.* [18].

A. Case I: Interaction of two particles in a parabolic confinement well

Although the particle interaction is known to include a wake potential, we can make qualitative predictions about the system by considering only the Hamiltonian part of the particle interaction. In the parabolic potential well,

$$V=V_D(\Delta x,\Delta z)+\frac{1}{4}M\omega_\rho^2\Delta x^2+\frac{1}{4}M\omega_z^2\Delta z^2.$$

If we assume that both Δx and Δz are nonzero and that the system is in equilibrium, then

$$\frac{Q_d^2}{4\epsilon_0}\frac{e^{-\kappa r}}{r^3}(1+\kappa r)\Delta x=\frac{1}{2}M\omega_\rho^2\Delta x,$$

$$\frac{Q_d^2}{4\epsilon_0}\frac{e^{-\kappa r}}{r^3}(1+\kappa r)\Delta z=\frac{1}{2}M\omega_z^2\Delta z.$$

It can be shown from the above equalities that the intermediate angles are stable only for $\omega_\rho=\omega_z$, otherwise, the system has only stable equilibria with the dust particles aligned horizontally or vertically (Fig. 2). This result should not be affected by the introduction of a wake charge since the wake acts to destabilize nonvertical particle arrangements. This was indeed shown by Lampe *et al.* [18], who gave an exact solution for two particles in the presence of the wake potential for a biquadratic confining potential.

If the vertical confinement exceeds the radial confinement, there are two degenerate equilibria with the particles aligned horizontally. As radial confinement is increased, the energies of the horizontal equilibria increase, becoming saddle points for $\omega_\rho/\omega_z>1$ at which point two new equilibria emerge in vertical alignment. The introduction of the wake provides the opportunity for metastable states in both horizontal and vertical alignment. This was also found by Lampe *et al.* [18], who used an effective energy to distinguish between ground and metastable states. Generally, ground and metastable states can be distinguished by their potential energy. If there is no Hamiltonian, there is still a way to determine the "potential energy″ as an integral of the interaction force [21] along the line between the pair of particles (i.e., alongside the interaction force). Although not the potential energy in the strict Hamiltonian sense, such introduced characteristics still allow us to determine ground and metastable states, as, e.g., in the theory of void formation [22]. The appearance of metastable void states can be seen in Fig. 8 of Ref. [22] where such defined energy was analyzed. The transition from the horizontal to the vertical alignments occurs at a reduced confinement ratio $\omega_\rho/\omega_z<1$ due to the reduced symmetry of the system caused by wake.

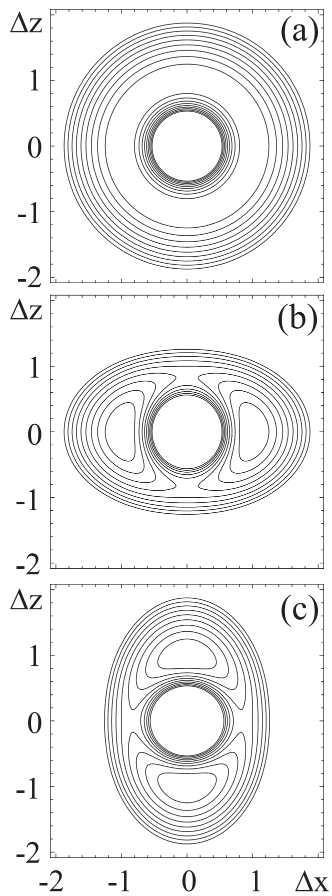

FIG. 2. Potential energy contours for two particles trapped in a parabolic well with (a) $a_2 = b_2$, (b) $a_2 < b_2$, and (c) $a_2 > b_2$. Where $a_2 = \frac{1}{2}M\omega_\rho^2$ and $b_2 = \frac{1}{2}M\omega_z^2$.

B. Case II: Interaction of two particles in a nonparabolic well

If instead of the parabolic approximation we consider the confinement potential up to fourth order,

$$V_{\mathrm{conf}} = \frac{1}{2}a_2\Delta x^2 + \frac{1}{3}a_3\Delta x^3 + \frac{1}{4}a_4\Delta x^4 + \frac{1}{2}b_2\Delta z^2 + \frac{1}{3}b_3\Delta z^3 + \frac{1}{4}b_4\Delta z^4$$

and employ the equilibrium condition $\partial V/\partial\Delta x = \partial V/\partial\Delta z = 0$, then

$$a_2 + a_3\Delta x + a_4\Delta x^2 = b_2 + b_3\Delta z + b_4\Delta z^2.$$

First, consider parabolic confinement with a cubic anharmonicity in the vertical direction ($b_3 \neq 0$) since the confining potential can have asymmetry in the z direction only (the potential is cylindrically symmetric and we put $a_3 = 0$). In order for this to be the case, the particles must have an asymmetry or the cubic terms will vanish [5]. Asymmetrical grain charging due to differing grain charge gradients $\partial Q_d/\partial z$ is one situation in which cubic terms may arise. If the coefficients of the parabolic confining terms are equal, then the only equilibria are saddle points located above and to the side of the origin; see Fig. 3. If $a_2 \neq b_2$, there are two degenerate equilibria which lie on the line $\Delta z = (a_2 - b_2)/b_3$. This suggests that the order parameter Δz will change continuously as the radial confinement strength is varied.

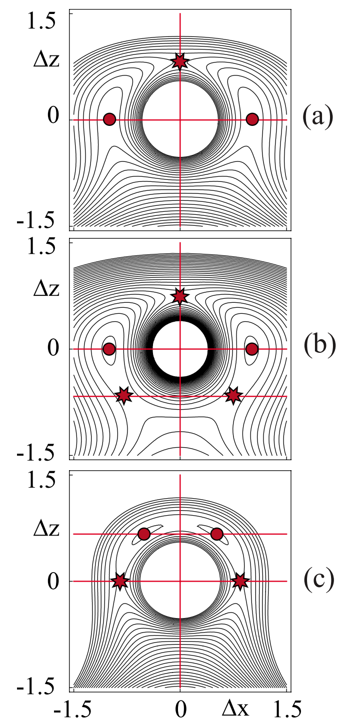

FIG. 3. (Color online) Potential energy contours for two particles trapped in a well with a cubic anharmonicity in the vertical direction ($a_3 = 0$) and (a) $a_2 = b_2$, (b) $a_2 < b_2$, and (c) $a_2 > b_2$. Circles represent stable equilibria and stars indicate saddle points.

Now consider symmetrical confinement $a_3 = b_3 = 0$ with nonzero quartic coefficients a_4 and b_4 [18]. For $\omega_\rho = \omega_z$, there are four equilibria that lie on an X in the $(\Delta x, \Delta z)$ plane,

$$\Delta z = \pm\sqrt{\frac{a_4}{b_4}}|\Delta x|, \tag{2}$$

and when $\omega_\rho \neq \omega_z$ they lie on the hyperbola

$$\Delta z = \pm\sqrt{\frac{a_2 - b_2 + a_4\Delta x^2}{b_4}}.$$

Note that the direction of the transverse axis of this hyperbola depends on the sign of $a_2 - b_2$ (Fig. 4). The presence of multiple equilibria suggests that jumping is theoretically possible. If the system remains in one of the equilibria, however, its position should change continuously with the confinement strength.

This also provides a useful diagnostic tool for determining the nonparabolicity of the confinement. When the radial and vertical confinements are equal, the dust particles should make an angle of $\tan\alpha = \sqrt{a_4/b_4}$ with the horizontal.

III. DETERMINATION OF THE ION WAKE CHARGE

For equimass particles near the horizontal plane, the particle-particle and wake-particle interactions can be approximated by a Hamiltonian system of the form $\mathcal{H}_{\mathrm{eff}} = T + V_{\mathrm{eff}}$ [7,23],

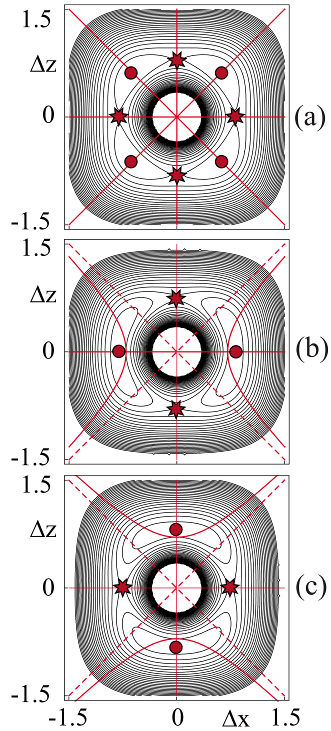

FIG. 4. (Color online) Potential energy contours for two particles trapped in a well with quartic anharmonicity ($a_4=b_4$) with (a) $a_2=b_2$, (b) $a_2>b_2$, and (c) $a_2<b_2$. Circles represent stable equilibria and stars indicate saddle points.

$$V_{\mathrm{eff}}=Q_d\{\Phi_D(\Delta x,\Delta z)+[\Phi_w(\Delta x,\Delta z)+\Phi_w(\Delta x,-\Delta z)]/2\} + V_{\mathrm{conf}}(\Delta x,\Delta z), \tag{3}$$

where the factor of $1/2$ has been included to account for the asymmetry of the particle-wake interaction. In this approximation, the stability of the horizontal alignment to vertical oscillations can be determined from the sign of the coefficient multiplying Δz^2 in the expansion of V_{eff} about $\Delta z=0$. The criterion for instability is that

$$\left(\frac{\omega_\rho}{\omega_z}\right)^2=1-\frac{Q_dQ_w}{2\epsilon_0 M\omega_z^2}\frac{e^{-\kappa\Delta_w}}{\Delta_w^5}(3\ell^2+3\ell^2\kappa\Delta_w+\ell^2\kappa^2\Delta_w^2),$$

where Δ_w is the distance from each dust particle to the other particle's wake and ω_ρ and ω_z denote the angular frequencies of a lone particle oscillating radially at constant height and vertically at the height of the center of mass of the two-particle system, respectively. Note that ω_z does not correspond to the vertical oscillation frequency of a lone particle in the sheath since the center of mass lies below the minimum of the confinement well.

Using experimental data for the critical frequencies, it is possible to make an estimation of the ion wake charge in these experiments. A surface of marginal stability can be generated by sampling a subset of points in the space of parameters $(\omega_\rho,\omega_z,Q_w)$. Projection of this surface onto the (ω_ρ,ω_z) plane reveals a set of isocontours corresponding to the critical instability at constant wake charge (Fig. 5). By

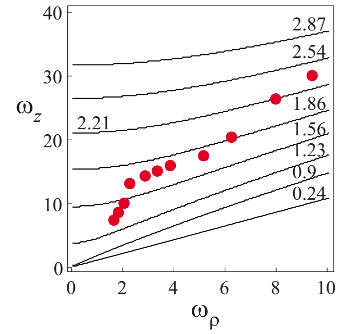

FIG. 5. (Color online) Wake charge isocontours showing the surface of marginal stability over the (ω_ρ,ω_z) plane. The units on the axes are rad s^{-1}.

superimposing the experimental points on the horizontal to vertical transition, it is possible to provide upper and lower bounds for the wake charge.

IV. THE VERTICAL TO HORIZONTAL TRANSITION

In addition to the horizontal alignment instability, the vertically aligned particles can destabilize resulting in a vertical to horizontal transition. In vertical alignment, the system cannot be assumed to behave as Hamiltonian due to the strongly asymmetric particle-wake interaction, which is only communicated downward. Note that for a biquadratic well the problem solved in Ref. [18] used the assumption that the system can be represented by an effective potential. Here, we employed a numerical search in (z_1,z_2) space for stationary solutions of the equations of motion for both dust particles. The currently understood theory of how the particles make their way from vertical to horizontal alignment involves continuous upward motion of the lower particle due to the shifting position of the confinement well, followed by a symmetry-breaking transition where the lower particle jumps directly to the horizontal plane.

The existence of this discontinuous vertical particle motion prior to the vertical to horizontal transition has not been explained. A qualitative explanation of this phenomenon can be based on an analogy with superposition of energy wells. In general, the vertical alignment can be classified into two subregimes. At low confinement strengths, the wake dominates and there is just one fixed point corresponding to the situation when the lower particle is directly inside the ion focus. At the other extreme of strong confinement, the wake-induced equilibrium disappears and the particle lies in the minimum of the confinement potential well. At moderate confinement strengths, two stable fixed points exist, necessarily separated by a third, unstable fixed point. Sweeping the control parameter ω_z in the reverse direction can cause the stable wake and unstable attractor to collide and annihilate each other in a global saddle node bifurcation, thus leading to a jump in the vertical position of the lower particle. The final destabilization of the vertical equilibrium initiates the vertical to horizontal transition, modeled also by Lampe *et al.* [18].

Discontinuities in the interparticle separation order parameter have also been observed prior to the second stage,

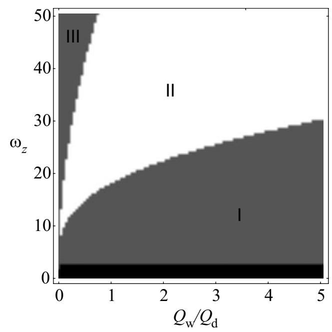

FIG. 6. Number of stable equilibria for two particles in vertical alignment as functions of Q_w and ω_z ($\ell/\lambda_D=3.5$). Region II corresponds to multiple equilibria and regions I and III correspond to a single equilibrium. The black region indicates areas where the model becomes numerically unstable. The units on the vertical axis are rad s^{-1}.

suggesting that the vertical to horizontal transition involves an additional critical phenomenon in which two vertically aligned equilibria—one due to the confinement well and the other due to perturbation of positive ions flowing in the sheath by the upper particle—merge to from a single equilibrium (see Fig. 6). Although the system is strictly speaking non-Hamiltonian, consideration on the basis of an effective potential energy $V_{\mathrm{eff}}=V_D+V_w+V_{\mathrm{conf}}$ can be used to explain the regimes as a superposition of two energy wells. Under some conditions the wells overlap creating a single potential minimum. Under another set of conditions the wells are spatially distinct, resulting in two energy minima separated by an unstable maximum.

When analyzing different regimes, a question on the real charge accumulated in the wake can appear. On a first approach, it might look reasonable to assume that the wake charge is less than the particle charge. Indeed, molecular dynamics simulations [17] show that $Q_w \sim 0.5-0.7Q_d$. However, these simulations do not include numerous processes taking place in discharge, in particular the presence of an ionization source strongly influencing all plasma parameters including the ion flux. Roughly, the total charge accumulated in the wake can be estimated as the ion flux multiplied by the focusing cross section. Taking into account recent developments in the theory of ion scattering [24] showing that the actual cross section may be enhanced due to large-angle scattering, and complex nonlinear dependence of the cross section on the ion flux due to dependence of the dust charge on the ion flux, it is actually difficult to set a limit on the wake charge. Therefore, we do not see compelling arguments that the case $Q_w > Q_d$ is totally excluded, and we included that possibility in Fig. 6. Note that for larger wake charges $Q_w > Q_d$ the region of a single equilibrium III almost disappears even for large enough ω_z.

V. CONCLUSION

We discussed the possible equilibria of a dust molecule in a variety of confinement potentials. Our analysis suggests that the parabolic potential well admits only horizontal and vertical stable dust alignments which is in agreement with an earlier prediction [18]. We also suggest that continuous changes in particle alignment are possible not just with a quartic potential well, but also with a cubic one due to particle asymmetry and variation of grain charge with elevation in the sheath.

The horizontal alignment instability at the critical confinement $\omega_{\rho,c}$ provides a diagnostic tool for determining the wake parameters. This will require accurate determination of the resonant frequencies of oscillation at the critical point. To our knowledge, no experiments have yet been performed in which the critical confinement frequencies for the horizontal instability of two identical dust particles were accurately measured. Data do exist, however, for the horizontal to vertical transition of two unequal mass particles [8]. This suggests the opportunity for an experiment to be performed with two identical particles with accurate control over the discharge parameters. Ultimately, however, the plasma parameters may be too sensitive to changes in the dc bias voltage on the radial confinement electrode to keep the wake sufficiently constant. More precise measurement of the wake may be possible by observing the laser-induced transition which was performed with two particles [25], since this guarantees constancy of the discharge parameters. In summary, the Hamiltonian approach can provide deep insights into the structure and stability of the dust molecule, and allows further elucidation of the wake and discharge parameters.

Finally, we note that dynamics of larger ensembles of dust particles is more complicated and can differ from the studied simplest case of a two-particle dust molecule. In particular, larger N-particle dust clusters can demonstrate a variety of stable and metastable configurations [1,26], with transitions between them [27]. Moreover, plasma absorption on dust [28] can affect interaction of dust particles, leading to long-range attractive and/or repulsive forces [2,29] which can be of collective character (such as collective attraction) [26]. Applicability of the Hamiltonian approach for these larger systems in the sheath region should be considered by taking into account not only the external confining potential but the effects of plasma fluxes on dust as well as inhomogeneities of the plasma and dust distributions for a particular system.

ACKNOWLEDGMENT

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APPENDIX: DERIVATION OF CRITICAL CONFINEMENT RATIO

Due to the horizontal reflection plane, the effective potential has a Landau expansion in the order parameter Δz,

$$V_{\mathrm{eff}} = a_0 + a_2 \Delta z^2 + \cdots ,$$

where

$$a_2 = \frac{1}{2} \left. \frac{\partial^2 V_{\text{eff}}}{\partial \Delta z^2} \right|_{\Delta z=0} = -\frac{Q_d^2}{8\epsilon_0} \frac{e^{-\kappa\Delta_d}}{\Delta_d^3}(1+\kappa\Delta_d) - \frac{Q_d Q_w}{8\epsilon_0} \frac{e^{-\kappa\Delta_w}}{\Delta_w^5} \times[-\kappa\Delta_w^3 + (\ell^2\kappa^2 - 1)\Delta_w^2 + 3\ell^2\kappa\Delta_w + 3\ell^2] + \frac{1}{4}M\omega_z^2$$

$$= \frac{1}{2}Q_d \frac{\partial\Phi_D(\Delta x,0)}{\partial\Delta x}\frac{1}{\Delta x} + \frac{1}{2}Q_d\frac{\partial\Phi_w(\Delta x,0)}{\partial \Delta x}\frac{1}{\Delta x} - \frac{Q_d Q_w}{8\epsilon_0}\frac{e^{-\kappa\Delta_w}}{\Delta_w^5}(\ell^2\kappa^2\Delta_w^2 + 3\ell^2\kappa\Delta_w + 3\ell^2) + \frac{1}{4}M\omega_z^2$$

$$= -\frac{Q_d Q_w}{8\epsilon_0}\frac{e^{-\kappa\Delta_w}}{\Delta_w^5}(\ell^2\kappa^2\Delta_w^2 + 3\ell^2\kappa\Delta_w + 3\ell^2) + \frac{1}{4}M\omega_z^2 - \frac{1}{4}M\omega_\rho^2.$$

When the coefficient vanishes, we have

$$\left(\frac{\omega_\rho}{\omega_z}\right)^2 = 1 - \frac{Q_d Q_w}{2\epsilon_0 M\omega_z^2}\frac{e^{-\kappa\Delta_w}}{\Delta_w^5}(\ell^2\kappa^2\Delta_w^2 + 3\ell^2\kappa\Delta_w + 3\ell^2).$$

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